About one long-range contribution to $K^+ \to \pi^+\ell^+\ell^-$ decays

Peter Lichard*

Department of Physics and Astronomy, University of Pittsburgh, Pittsburgh, Pennsylvania 15260; and Institute of Physics, Silesian University, 746-01 Opava, Czech Republic

Abstract

We investigate the mechanism of $K^+ \to \pi^+\ell^+\ell^-$ ($\ell = e,\, \mu$) decays in which a virtual photon is emitted either from the incoming K^+ or the outgoing π^+. We point out some inconsistencies with and between two previous calculations, discuss the possible experimental inputs, and estimate the branching fractions. This mechanism alone fails to explain the existing experimental data by more than one order-of-magnitude. But it may show itself by its interference with the leading long-range mechanism dominated by the a_1^+ and ρ^0 mesons.

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*On leave of absence from Department of Theoretical Physics, Comenius University, 842-15 Bratislava, Slovak Republic.

With the new data on the $K^+ \to \pi^+\ell^+\ell^-$ decays coming soon [1], thoughts about the possible mechanism(s) behind them are gaining steam. One candidate is virtual photon emission from the incoming K^+ and outgoing π^+; it was conjectured recently that its contribution to the decay amplitude may be important [2]. The purpose of this note is to scrutinize the role of this mechanism.

The matrix element of this mechanism was given in a paper by Vainshtein, Zakharov, Okun, and Shifman [3] and used in the same form by Bergström and Singer [4]. After changing to contemporary notations and to the $(+,-,-,-)$ metrics, the matrix element becomes

$$\begin{aligned}\mathcal{M} = {}& \frac{e^2 G_F}{\sqrt{2}}(f_{\pi^+}V_{ud})(f_{K^+}V_{us}^*)\frac{1}{m_K^2 - m_\pi^2}\left[\frac{m_K^2}{6}\langle r^2\rangle_{K^+} - \frac{m_\pi^2}{6}\langle r^2\rangle_{\pi^+}\right] \\ & \times (p_K + p_\pi)^\mu \ \overline{u}(p_-)\gamma_\mu v(p_+) \ . \end{aligned} \tag{1}$$

The formula we derive in this note differs from (1). Another formula exists in the literature [5], which differs from ours "only" in the multiplicative constant. Despite the fact that our formula differs more substantially from Eq. (1) than it does from that in [5], we consider that our formula is more closely related to Eq. (1) than to the latter in the underlying physics, as we will discuss later.

Because of these discrepancies we present the derivation of our formula in thorough detail. But let us first define, for completeness and later reference, the quantities which enter Eq. (1). G_F is the Fermi constant related to the weak coupling constant, g, and to the $W^\pm$ boson mass by $g^2 = 4\sqrt{2}G_F m_W^2$. The decay constant of the π^+ meson is defined by

$$\langle 0|\bar{d}(0)\gamma^\mu\gamma_5 u(0)|p\rangle_{\pi^+} = i \ f_{\pi^+} \ p^\mu \ .$$

The definition of the K^+ decay constant, f_{K^+}, is analogous. The Vs are the elements of the Cabibbo-Kobayashi-Maskawa matrix. Using the K^+ and π^+ mean lifetimes, the branching fractions of decays $\pi^+ \to \mu^+\nu_\mu$ and $K^+ \to \mu^+\nu_\mu$ given in [6], and the method of radiative corrections described there, we obtained the following values:

$$\begin{aligned}|f_{\pi^+}V_{ud}|^2 &= (1.6418 \pm 0.0010) \times 10^{-2}\ \mathrm{GeV}^2, \\ |f_{K^+}V_{us}|^2 &= (1.2471 \pm 0.0044) \times 10^{-3}\ \mathrm{GeV}^2,\end{aligned}$$

which we use below in numerical calculations. The mean square radii (MSR) are defined by means of the low $|t|$ expansion of the electromagnetic form factor

$$F(t) = F(0) + \frac{\langle r^2\rangle}{6}\ t + \cdots \ , \tag{2}$$

where $F(0) = 1$ (0) for charged (neutral) mesons. Finally, spinors u and v in Eq. (1) refer to the outgoing leptons.

Let us now derive our formula. When the electromagnetic interaction is switched off, the dynamics of the system containing charged pions, kaons, and weak-gauge bosons is described by the effective Lagrangian[1]

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[1] In [7] we used a simpler effective Lagrangian with a contact weak interaction between mesons to

$$\mathcal{L}_0 = \mathcal{L}_{\text{free}} + \mathcal{L}_{\text{weak}}, \tag{3}$$

where $\mathcal{L}_{\text{free}}$ is the well-known Lagrangian of the free $\pi^\pm$, $K^\pm$, and $W^\pm$ fields, and

$$\mathcal{L}_{\text{weak}} = -i\frac{g}{2\sqrt{2}}W^\dagger_\mu \left(f_{\pi^+}V^*_{ud}\partial^\mu\phi_\pi + f_{K^+}V^*_{us}\partial^\mu\phi_K\right) + \text{H.c.} \tag{4}$$

After turning on the electromagnetic interaction by the minimal substitution $\partial^\mu \to \partial^\mu + ieA^\mu$, the Lagrangian acquires additional terms and becomes

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_\gamma + \mathcal{L}_{\gamma\gamma}\ .$$

The last part contains the product of two electromagnetic field operators and does not operate in our case. The one-photon part contains two terms

$$\mathcal{L}_\gamma = \mathcal{L}_{\gamma,\text{free}} + \mathcal{L}_{\gamma,\text{weak}}.$$

The former was induced from the free part of the Lagrangian (3), and the latter from its weak interaction part (4). They are given by

$$\begin{aligned}\mathcal{L}_{\gamma,\text{free}} &= ieA^\mu \sum_{P=\pi,K}\left[\left(\partial_\mu\phi^\dagger_P\right)\phi_P - \phi^\dagger_P\partial_\mu\phi_P\right]\ , \\ \mathcal{L}_{\gamma,\text{weak}} &= \frac{eg}{2\sqrt{2}}A^\mu W^\dagger_\mu \left(f_{\pi^+}V^*_{ud}\phi_\pi + f_{K^+}V^*_{us}\phi_K\right) + \text{H.c.}\end{aligned}$$

The amplitude of the (virtual) photon emission from $W^\pm$ is suppressed by a factor of $(m_K/m_W)^2$, and hence, the corresponding term in $\mathcal{L}_{\gamma,\text{free}}$ is not shown.

To proceed further, we need to determine the interaction Hamiltonian. We can show that the relation

$$\mathcal{H}_{\text{int.}} = -\mathcal{L}_{\text{weak}} - \mathcal{L}_\gamma - \mathcal{L}_{\gamma\gamma}$$

holds, up to non-covariant terms which vanish when the matrix element is calculated between the physical states [8,9]. After deriving the Feynman rules in momentum space, we modified them by multiplying the electric charge of mesons by the corresponding form factors $F_P(q^2)$ ($P = \pi^+,\ K^+$), thereby accounting for the π^+ and K^+ internal structure. As a result, we obtain the following vertex (junction) factors, starting with the well-known electromagnetic vertex induced from the free Lagrangian

- $-ieF_P(q^2)(p_a + p_b)^\mu$ for the $P \to P\gamma$ vertex. Here, p_a (p_b) is the four-momentum of the incoming (outgoing) meson, $q = p_a - p_b$ is the (virtual) photon momentum, and μ is the index connected with the photon line;
- $-ig/(2\sqrt{2})\ f_{K^+}V^*_{us}\ p^\mu$ for the $K^+ \to W^+$ junction;

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show that, for the point-like mesons, the matrix element considered here vanishes. We checked that the simpler Lagrangian gives the same results as presented here, but prefer to be more rigorous.

- $-ig/(2\sqrt{2})\ f_{\pi^+} V_{ud}\ p^\mu$ for the $W^+ \to \pi^+$ junction;
- $ieg/(2\sqrt{2})\ F_{K^+}(q^2)\ f_{K^+} V_{us}^*\ g^{\mu\nu}$ for the $K^+ \to W^+\gamma$ vertex;
- $ieg/(2\sqrt{2})\ F_{\pi^+}(q^2)\ f_{\pi^+} V_{ud}\ g^{\mu\nu}$ for the $W^+ \to \pi^+\gamma$ vertex.

The contributions to the matrix element of the $K^+ \to \pi^+\ell^+\ell^-$ decay are depicted in Fig. 1. Their evaluation leads to

$$\begin{aligned} \mathcal{M} = {} & \frac{e^2 G_F}{2\sqrt{2}} (f_{\pi^+} V_{ud})(f_{K^+} V_{us}^*)\ \frac{m_K^2 + m_\pi^2}{m_K^2 - m_\pi^2}\ \frac{F_{K^+}(t) - F_{\pi^+}(t)}{t} \\ & \times (p_K + p_\pi)^\mu\ \overline{u}(p_-)\gamma_\mu v(p_+)\ , \end{aligned} \tag{5}$$

where $t = (p_K - p_\pi)^2$. When writing (5) we took advantage of the fact that the contraction of $(p_K - p_\pi)^\mu$ with the lepton term vanishes. Finally, if we ignore possible contributions to the matrix element from other mechanisms, we get the following formula for the differential decay rate in the $\ell^+\ell^-$ mass $M = \sqrt{t}$:

$$\begin{aligned} \frac{d\Gamma_{K^+\to\pi^+\ell^+\ell^-}}{dM} = {} & \frac{G_F^2 \alpha^2}{48\pi m_K^3}\, |f_{\pi^+} V_{ud}|^2\, |f_{K^+} V_{us}|^2 \left(\frac{m_K^2 + m_\pi^2}{m_K^2 - m_\pi^2} \right)^2 \\ & \times \lambda^{3/2}(m_K^2, m_\pi^2, t)\ \sqrt{t - 4m_\ell^2} \left(1 + \frac{2m_\ell^2}{t} \right) \left| \frac{F_{K^+}(t) - F_{\pi^+}(t)}{t} \right|^2\ , \end{aligned} \tag{6}$$

where $\lambda(x,y,z) = x^2 + y^2 + z^2 - 2xy - 2xz - 2yz$. All quantities on the left-hand side of Eq. (6) are well known, except the form factors. We need their values in the time-like region below the physical thresholds, where they are inaccessible to direct measurement. Only for $F_{\pi^+}(t)$ a small part

$$4m_\pi^2 < t < (m_K - m_\pi)^2 \tag{7}$$

of the $K^+ \to \pi^+\ell^+\ell^-$ t-interval lies in the physical region of the e^+e^- annihilation. To determine the form factors in the region of our interest, one has to combine the experimental information about them in both the space-like and time-like regions with their analytic properties. This was undertaken, *e.g.*, in Ref. [10]. Unfortunately, in the small-t region, both the π^+ and K^+ form factors differ only a little from unity, which greatly increases the relative error of their difference. Therefore, one must seek alternative methods for determining the difference in form factors in (6); we suggest a few below. Two are based on the low $|t|$ expansion of the electromagnetic form factor (2). Most of our methods rely on the vector meson dominance (VMD) description of the form factors of pions and kaons. So, first, we sketch the necessary VMD formulas.

In VMD, the form factors are described by assuming that the pseudoscalar mesons couple to the photon via the vector meson resonances ρ^0, ω, and ϕ. Let the functions $V(t)$ ($V = \rho$, ω, ϕ), normalized by the condition $V(0) = 1$, describe the contributions of individual resonances to the pseudoscalar meson form factors. The normalization conditions imposed on the form factors, together with the assumptions about the isospin invariance of the strong vertices, lead to the formulas

$$
\begin{aligned}
F_{\pi^+}(t) &= \rho(t)\ , \\
F_{K^+}(t) &= \ \frac{1}{2}\rho(t) + c\,\omega(t) + \left(\frac{1}{2} - c\right)\phi(t)\ , \\
F_{K^0}(t) &= -\frac{1}{2}\rho(t) + c\,\omega(t) + \ \left(\frac{1}{2} - c\right)\phi(t)\ , \qquad (8) \\
F_{K^0}(t) &= F_{K^+}(t) - F_{\pi^+}(t)\ . \qquad (9)
\end{aligned}
$$

In principle, the constant c can be fixed by assuming the (flavor) SU(3) invariance. Because the latter is violated, we will, instead, consider c a phenomenological parameter, and fix its value by experimental data. But before proceeding further, we must specify the functions $V(t)$. For narrow resonances ω and ϕ, which, in addition, do not have any open decay channels in our t-range, we can use the prescription

$$
V(t) = \frac{m_V^2}{m_V^2 - t}\ , \qquad (10)
$$

which originates from the free-vector-particle propagator, taking into account that the term in its numerator containing four-momenta does not contribute.

But the situation with the ρ^0 is more complicated. This is evident from the fact that using the simple formula (10) for $\rho(t)$ gives the MSR of π^+ equal to (0.3940 ± 0.0008) fm^2, which disagrees with the experimental findings that give (0.439 ± 0.008) fm^2 [11]. We will, therefore, use the form that properly accounts for the dynamics of resonances (see, *e.g.*, [12])

$$
\rho(t) = \frac{m_\rho^2(0)}{m_\rho^2(t) - t - i m_\rho \Gamma_\rho(t)}\ , \qquad (11)
$$

where $m_\rho^2(t)$ is the running mass squared, m_ρ is the resonant mass, and $\Gamma_\rho(t)$ is the variable total width. The latter vanishes in our kinematic range, with the exclusion of the small region already mentioned (7). We take the function $m_\rho^2(t)$ from our recent work [12]; this gives the π^+ MSR of (0.446 ± 0.006) fm^2, in good agreement with data.

Below, we briefly describe six methods for evaluating the form-factor difference, which we need to be able to calculate the branching fraction (6) of the $K^+ \to \pi^+\ell^+\ell^-$ decays. The corresponding results for both the e^+e^- and $\mu^+\mu^-$ modes are shown in Table 1.

1. Both $F_{\pi^+}(t)$ and $F_{K^+}(t)$ are taken from Ref. [10]. We do not have any way to assess the errors, so the results are shown with the "approximate" sign.

2. Formula (9) is used, together with the parametrization of $F_{K^0}(t)$ from [10]. The same caveat as above applies.

3. Low-t expansion (2) is used with

$$
\langle r^2\rangle_{\pi^+} - \langle r^2\rangle_{K^+} = (0.100 \pm 0.045)\ \text{fm}^2\ , \qquad (12)
$$

taken from Ref. [13].

4. The formula (9) and parametrization (2) for $F_{K^0}(t)$ are used with

$$\langle r^2 \rangle_{K^0} = -(0.054 \pm 0.026)\ \mathrm{fm}^2. \tag{13}$$

   This value was determined in [14] by measuring the coherent regeneration of K_S^0s by atomic electrons; the method was proposed by Zel'dovich [15].

5. Formulas (8) and (9) are used with $c = 0.07 \pm 0.29$, which was determined by matching the MSR difference (12).

6. As above, but the MSR of K^0 (13) was matched with $c = 0.36 \pm 0.17$.

Of all of those methods, we believe that the last two are the most reliable. As we argued, the calculation based directly on the form factors (method 1) is hampered by the large relative errors in their difference. Method 2, using the K^0 form factor, is probably more reliable, but similarly here, we cannot estimate the final error. Methods 3 and 4 lead to a constant form factor of the $K^+ \to \pi^+ \ell^+ \ell^-$ decay, defined below in Eq. (14), which is a bad approximation [16]. Our preference for methods 5 and 6 stems from the fact that the systematic errors of the VMD method are certainly smaller than the errors of the data we use as input. Thus, the extrapolation from $t = 0$, where we fixed parameter c [see Eq. (8)], to higher values of t is well defined. Morever, c determines the relative weight of the ω and ϕ contributions, which do not differ greatly in our region of t. So, the value of c is not critical.

The branching fractions are not the only quantities that can be compared to experimental data. The decay form factor $f(t)$, which also is very important, is defined through the matrix-element parametrization

$$\mathcal{M} = C\ f(t)\ \ (p_K + p_\pi)^\mu\ \ \overline{u}(p_-)\gamma_\mu v(p_+) \tag{14}$$

and the normalization condition $f(0) = 1$. Comparing Eqs. (5) and (14) reveals that the mechanism considered here exhibits a form factor

$$f(t) = \frac{1}{t} \frac{F_{K^+}(t) - F_{\pi^+}(t)}{F'_{K^+}(0) - F'_{\pi^+}(0)}\ . \tag{15}$$

In Fig. 2, we plot the absolute value of this form factor assuming the VMD relations (8) and (9) with $c = 0.36$ (method 6), and of the form factor calculated for the long-range mechanism dominated by the a_1^+ and ρ^0 mesons [7,12]. What is finally observed experimentally is the form factor belonging to the superposition of those two mechanisms. This most important result is depicted by the solid line. Because the normalisation of the meson-dominance matrix element is yet a little uncertain, see discussion in [7], we considered its multiplicative constant a free parameter. Its value was chosen to get the correct branching fraction for the e^+e^- mode (2.74×10^{-7}, [6]). For comparison, we also show the linear parametrization of the form factor used by experimentalists

$$f(t) = 1 + \lambda \frac{t}{m_\pi^2}\ ,$$

where $\lambda = 0.105$ [16]. Taking into account the experimental errors of λ, which are 0.035 (stat.) and 0.015 (syst.) [16], we can show that the solid curve is compatible with

the published experimental results. However, the preliminary data of the BNL-E865 collaboration [1] indicates a steeper slope, which may be a problem for the meson dominance model, even when it is supplemented with the mechanism considered in this note.

Having fixed the form factor that belongs to the superposition of the two mechanisms mentioned above, we can evaluate the $\mu^+\mu^-/e^+e^-$ branching ratio. It comes out as

$$\frac{B(K^+ \to \pi^+\mu^+\mu^-)}{B(K^+ \to \pi^+e^+e^-)} = 0.248 \pm 0.002.$$

The quoted error reflects the uncertainties of the parameter c and of the ρ^0 running mass squared [12] in our t-region. Using the recommended value $(2.74 \pm 0.23) \times 10^{-7}$ of the branching fraction of the e^+e^- mode [6], we end up with the prediction

$$B(K^+ \to \pi^+\mu^+\mu^-) = (6.8 \pm 0.6) \times 10^{-8}.$$

Let us compare now our formula for the matrix element and the numerical results with those of previous studies [3,5].

Our matrix element (5) differs from that derived in [3], see Eq. (1), even after the form-factor difference in our formula is expressed in terms of the mean radii squared, using Eq. (2). We could not locate the source of the discrepancy. To see how serious this discrepancy is from a pragmatic point of view, we calculated the absolute value of the matrix element ratio using the mean square radii of π^+ and K^+ of 0.44 fm^2 and 0.34 fm^2, respectively [11,13]. The result is

$$\left|\frac{\mathcal{M}(\text{this work})}{\mathcal{M}(\text{Ref. [3]})}\right| = \frac{m_K^2 + m_\pi^2}{2}\left|\frac{\langle r^2\rangle_{K^+} - \langle r^2\rangle_{\pi^+}}{m_K^2\langle r^2\rangle_{K^+} - m_\pi^2\langle r^2\rangle_{\pi^+}}\right| \approx 0.18 \ .$$

A formula for the matrix element of the same "inner-bremsstrahlung" mechanism was proposed also in [5]. It contains a simple difference of the π^+ and K^+ electromagnetic form factors, in agreement with our findings and in disagreement with (1). However, the same form was obtained by different means. The authors of [5] simply assumed that the $K^+ \to \pi^+$ transition amplitude was a constant, whereas in [3], and also in our approach, this amplitude is proportional to the momentum squared. The latter is equal to the mass squared of that of the two mesons which is on the mass shell. In our approach the form factor difference results from the interplay between the momentum-dependent transition amplitude and the contributions to the matrix element that originate in the weak-interaction part of the Lagrangian (3).

The multiplicative constant in the matrix element presented in [5] is fixed, through a chain of reasoning, by the data on the $K \to \pi\pi$ and $K^+ \to \mu^+\nu_\mu$ decay rates. This shows that the physics assumed to lie behind the $K^+ \to \pi^+$ transition is different from that here and in Ref. [3], where the former source of information is replaced by the $\pi^+ \to \mu^+\nu_\mu$ decay rate.

What concerns the magnitude of the matrix element here and in [5], their estimate

$$\left|\langle\pi^+|H_w|K^+\rangle\right| \approx 3.9 \times 10^{-8}\ \text{GeV}^2$$

should be compared to ours

$$\frac{G_F}{2\sqrt{2}} \left| f_{\pi^+} V_{ud} f_{K^+} V_{us} \right| (m_K^2 + m_\pi^2) = 4.91 \times 10^{-9} \ .$$

The strikingly different magnitudes of the matrix element lead to different judgements about the role of the considered mechanism. In [5] it was deemed very important, with the matrix element twice as big as that required by experimental branching fraction for the e^+e^- mode. Here, we found (see Table 1) that this mechanism, if taken alone, leads to branching fractions that are at least one order-of-magnitude below the experimental values.

It was shown in [7] that the a_1/ρ-meson dominance makes the leading contribution to the matrix element of the $K^+ \to \pi^+\ell^+\ell^-$ decays. Now we have shown that the matrix element of the mechanism considered here is several times smaller (4-8 times, if we quote the results of methods 5 and 6, which we trust the most). Neverthless, its interference with the dominant mechanism somewhat improves the behavior of the form factor, which is a little too flat for the dominant mechanism alone [12].

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TABLES

TABLE I. Branching fractions of the $K^+ \to \pi^+ e^+ e^-$ and $K^+ \to \pi^+ \mu^+ \mu^-$ decays calculated using different inputs and compared to data [6]: (1) K^+ and π^+ form factors [10]; (2) K^0 form factor [10]; (3) MRS difference of K^+ and π^+ [13]; (4) MRS of K^0 [14]; (5) VMD form factor of K^0 with c fixed by the difference in K^+ and π^+ MRS [13]; (6) as in (5), but fixed by the K^0 MRS [14].

Method	1	2	3	4	5	6	Exp. data
$B(e^+e^-) \times 10^9$	≈ 1.3	≈ 22	10^{+11}_{-7}	$2.8^{+3.3}_{-1.0}$	14^{+13}_{-9}	$4.6^{+4.5}_{-2.9}$	274 ± 23
$B(\mu^+\mu^-) \times 10^9$	≈ 0.3	≈ 7.4	$1.9^{+2.1}_{-1.3}$	$0.54^{+0.65}_{-0.40}$	$4.0^{+3.5}_{-2.4}$	$1.5^{+1.2}_{-0.9}$	50 ± 10

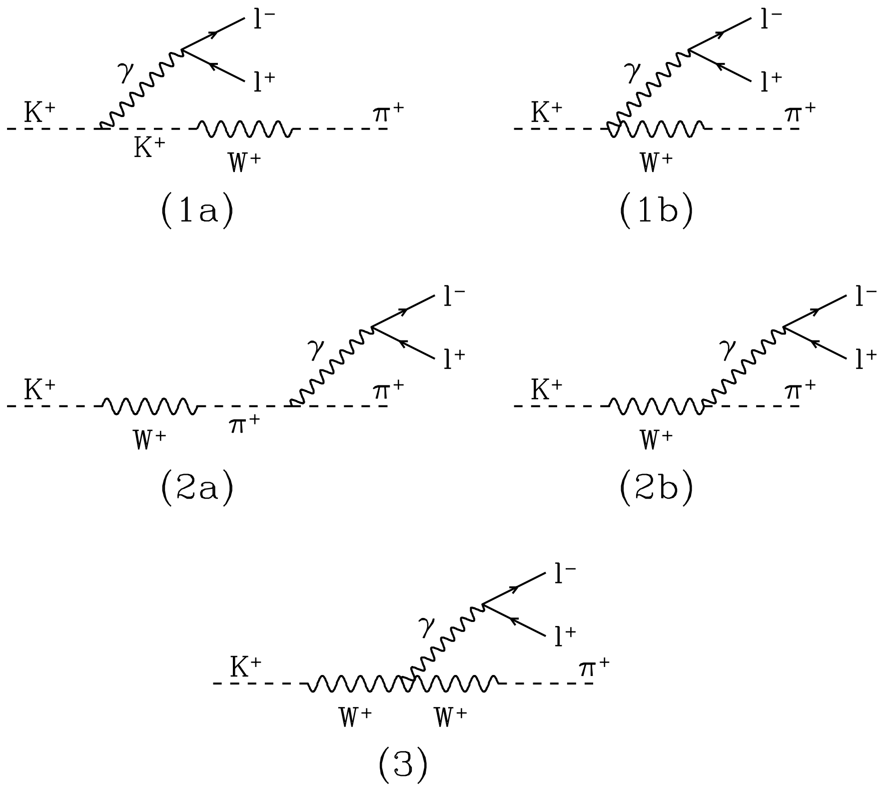

FIG. 1. Feynman diagrams contributing to the matrix element of the $K^+ \to \pi^+\ell^+\ell^-$ decay. Contributions (1a) and (1b) are proportional to the K^+ form factor, (2a) and (2b) to the π^+ form factor. Contributions (1a) and (2a) were generated from the free part of the Lagrangian, and (1b) and (2b) from its weak-interaction part. Contribution (3) is suppressed by an additional factor of $(m_K/m_W)^2$ and therefore is ignored.

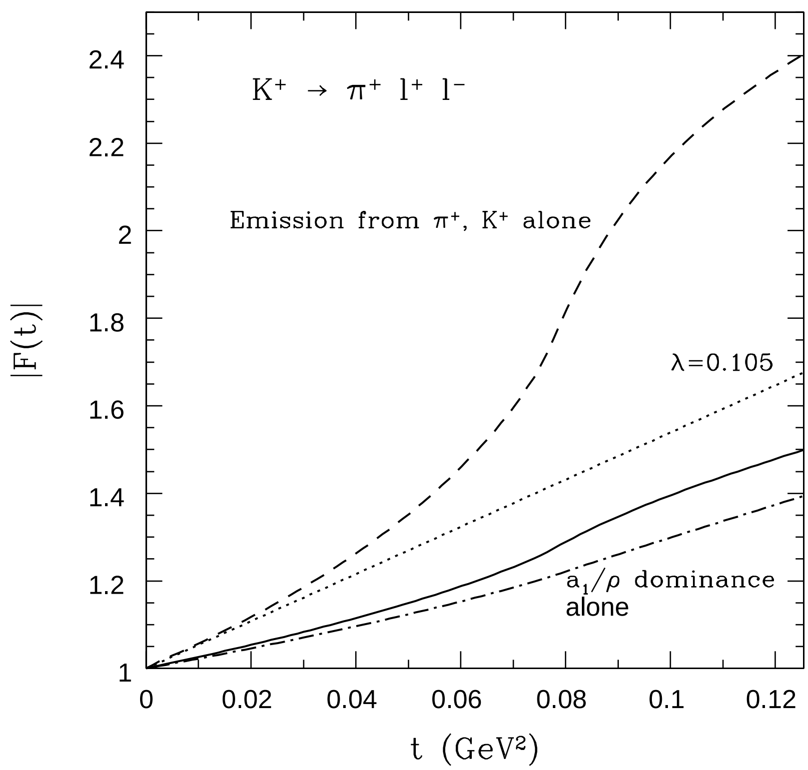

FIG. 2. Form factor of the decay $K^+ \to \pi^+\ell^+\ell^-$: The mechanism considered in this work taken alone (dashed line); The form factor coming from the a_1/ρ dominance [7,12] (dash-dotted); Superposition of the a_1/ρ dominance and the present mechanism (solid); The linear parametrization used to fit data [16] (dotted).